Geometric Phases in Magnetooptic Channel Waveguide Devices

M. Concepción Nistal, Jesús Liñares, and Daniel Baldomir

Abstract—**Geometric phases, generated by a coupling process between TE and TM polarizations through anisotropic chiral media—magnetooptic channel waveguides—are determined. In fact, many of the conventional phase and frequency shifters are based on this geometrical effect, nevertheless, the physical origin of these phase factors (spatial and temporal ones) has never been explained. In this work a physical interpretation, based on the topological phases theory, is given. Likewise, an integrated interferometer is proposed for both generating and checking these phases by changing the characteristics of the channel waveguides. The relationship between geometric factors and phase and frequency shifters is shown along the work.**

I. Introduction

GEOMETRIC phases in the cyclic evolution of many physical systems have been studied during the past few years extensively. Numerous publications containing several applications appeared in different fields such as, molecular physics [1], nuclear resonance [2], optics [3], neutrons [4], electromagnetic coupling [5] and so on. Originally, the geometric phase was introduced by Berry [6] for adiabatic processes; further, Aharonov and Anandan (AA) [7] have liberated it from the restriction to adiabatic evolution, and Samuel and Bhandari [8] have given a general setting for Berry's phase: the evolution of the system need be neither unitary nor cyclic.

Inside electromagnetic theory, geometric phases have been determined for N coupled electromagnetic wave amplitudes evolving in their projective Hilbert space [5]. This kind of system constitutes a general framework of geometric phases for electromagnetic waves, that is, it includes geometric factors generated by polarization coupling, grating coupling [9], multidirectional (in particular bidirectional) coupling [10] between planar or channel waveguides [5] and so on. On the other hand, from a dynamical point of view, magnetooptic (and anisotropic) channel waveguides have been proposed as waveguide isolators [11]. These components allow a compact integration of coherent devices in comunication systems. Thus, isolators, circulators, phase shifters, rotators, and so on, based on magnetooptic coupling, can be designed.

In this work we center our attention on the geometric phases generated by the electromagnetic coupling process between polarization states (TE-TM) caused by these anisotropic chiral media—magnetooptic channel waveguides. In fact, many magnetooptic devices can generate spatial and temporal phases whose origin (as it will be proven) is of topological nature, being the primary aim of this work to show how these phase factors can be generated and calculated. The main characteristic of these geometric phases (nondynamical phase) is that they take values which are not dependent on the length of the channel waveguide. For instance, a simple phase π was found for a pure Faraday rotation in an isotropic magnetooptic medium [12]. Here we present a generalization of this effect. First of all and for sake of simplicity we analyze a 90° rotor which generates a phase shift in the output state, after propagation along the channel waveguide. This geometric phase can be controlled by changing the characteristics of the channel waveguide (propagation constants and magnetooptic coupling coefficient). Then, we propose an interferometric system constituted by magnetooptic channel waveguides generating geometric phases. Two experiments are proposed, being the second one, from a theoretical point of view, related to an Aharonov–Bohm (AB) effect on the Poincare's sphere (PS).

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II. Dynamical Evolution

Let us consider an electromagnetic wave propagating through an anisotropic channel waveguide. We can describe the dynamical evolution in this system by a Schrodinger-type equation in a two-dimentional projective Hilbert space [5]

$$H|\varphi\rangle = i\partial_Z|\varphi\rangle \tag{1}$$

where z indicates the direction of propagation, H the relevant Hamiltonian of the system and $|\varphi\rangle^T = (\varphi_1, \varphi_2)^T$ (T indicating transpose), and φ_1 and φ_2 are the amplitudes of the TE and TM modes respectively. In our case, the relevant Hamiltonian can be written in a compact form as [5]

$$H = \mu_{ij} = \beta_{ij}\delta_{ij} + C\sigma_2 \tag{2}$$

where μ_{ij} is an effective tensor permeability with β_{11} and β_{22} being the TE and TM propagation constants, C the magnetooptic coupling coefficient and σ_2 the second Pauli's matrix.

By solving (1) we obtain after a long but straightforward calculation, the following expression for the vector $|\varphi\rangle$:

$$|\varphi\rangle^T = (\varphi_1, \varphi_2)^T \exp(i\alpha z) \tag{3}$$

with α the halfsum of the TE and TM propagation constants and

$$\varphi_1 = \varphi_1(0)\left\{\cos(\gamma z) + i\left[\frac{(e_+ + e_-)}{(e_+ - e_-)}\right]\sin(\gamma z)\right\} + \varphi_2(0)\left\{\left[\frac{2}{(e_+ - e_-)}\right]\sin(\gamma z)\right\} \tag{4}$$

$$\varphi_2 = \varphi_1(0)\left\{\left[\frac{2}{(e_+ - e_-)}\right]\sin(\gamma z)\right\} + \varphi_2(0)\left\{\cos(\gamma z) - i\left[\frac{(e_+ + e_-)}{(e_+ - e_-)}\right]\sin(\gamma z)\right\} \quad (5)$$

vector $(\varphi_1(0), \varphi_2(0))$ represents the TE-TM components of the electromagnetic wave at the input plane $z = 0$; γ and β_- are given by the following expresions

$$\gamma = \sqrt{(\beta_-^2 + C^2)} \quad (6)$$

$$\beta_- = \frac{\beta_{11} - \beta_{22}}{2} \quad (7)$$

and $e\pm$ is related to the anisotropy of the medium through the relationship

$$e_\pm = \frac{\beta_-}{C} \pm \sqrt{\left[\left(\frac{\beta_-}{C}\right)^2 + 1\right]}. \quad (8)$$

Note that for $\beta_- = 0$, $e\pm = \pm 1$, and therefore a pure Faraday rotator is obtained.

III. Geometric Phases

In this section, we analyze the geometric properties of the electromagnetic TE-TM coupling through a magnetooptic channel waveguide. We must stress that the TE and TM propagation constants of the magnetoooptic channel waveguide—defined in the Hamiltonian equation (2)—are not very different, therefore the geometric factors derived in this section are of the AA type [7]; that is, the adiabatic evolution will not be invoked. Thus following the Aharonov–Anandan's criterion [7] to determine geometric phases for any closed circuit on the projective Hilbert space, the geometric factor can be evaluated by the following equation

$$\phi_G = \psi + \int_0^d \langle\varphi|H|\varphi\rangle dz \quad (9)$$

where d is the traveled space, Ψ is the full phase and the integral term is equal to minus the dynamical phase. Due to the topological nature of these factors they can be also expressed in a geometrical form as [6]

$$\phi_G = \pm\frac{1}{2}\Omega(L) \quad (10)$$

where L is the circuit followed by the vector $|\varphi\rangle$ on the PS (two-dimensional Hilbert space) and $\Omega(L)$ is the solid angle subtended by the circuit L from the origin of the PS.

Let us consider that the Hamiltonian given by (2) acts on the initial state $b(1,1)^T$ (point A on the sphere; see Fig. 1) where b is a normalization constant; then the state at a depth $z = d$ in the channel waveguide can be calculated from (4), (5). Thus, if the following relationship is fulfilled

$$\gamma = \frac{(2n+1)}{2d}\pi \quad (11)$$

then the orthogonal state $b(1,-1)^T$ is reached (point B in Fig. 1), where the vector $|\varphi\rangle$ has followed the geodesic indicated in Fig. 1. The circuit can be closed (in a geometrical

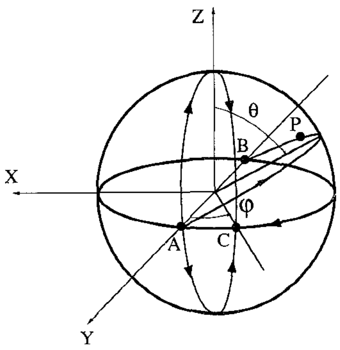

Fig. 1. Poincare's sphere showing the paths followed by the polarization state of the electromagnetic wave in this work.

way) by the geodesic along equator (slice circuit) [5]. From a physical point of view the circuit can be closed along equator by an isotropic channel waveguide. The geometric phase acquired by the state at point B can be obtained in the following way: first, we calculate the integral term in (9) (dynamical phase) by using (2)–(5), therefore

$$\int_0^d \langle\varphi|H|\varphi\rangle dz = -\frac{\alpha\pi}{2\gamma} \quad (12)$$

then, from (3)–(5), the full phase acquired by the state $b(1,-1)^T$ at point B can be obtained taking into account that the state $|\varphi\rangle$ can be rewritten as

$$|\varphi\rangle = b[B - iA](1,1)^T e^{i\alpha d} \quad (13)$$

where

$$A = \frac{e_+ + e_-}{e_+ - e_-} \qquad B = \frac{2}{e_+ - e_-} \quad (14)$$

which fulfills the following relationship

$$A^2 + B^2 = 1. \quad (15)$$

Now, we express the above relationships in such a way the parameters of the device can be related with the topological circuit on the Poincare's sphere (slice circuit; see Fig. 1) that is,

$$A = \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta = \frac{\beta_-}{\gamma} \quad (16)$$

$$B = \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta = \frac{C}{\gamma} \quad (17)$$

where θ is the angular extension of the slice circuit described by the state $|\varphi\rangle$. Now, inserting these equations in (13) we obtain

$$|\varphi\rangle = b\exp\left[i\left(\frac{\pi}{2} - \theta\right) + i\alpha\frac{(n+\frac{1}{2})}{\gamma}\pi\right](1,-1)^T. \quad (18)$$

Therefore, from this equation the full phase is given by

$$\psi = \left[\left(\frac{\pi}{2} - \theta\right) + \alpha\frac{(n+\frac{1}{2})}{\gamma}\pi\right]. \quad (19)$$

Finally, from (9), the geometric phase acquired by the state at point B on the Poincare's sphere is given by

$$\phi_G = \left(\frac{\pi}{2} - \theta\right). \tag{20}$$

Likewise, if we use (10) for the circuit described by the state $|\varphi\rangle$ on the PS, after a straightforward calculation, the solid angle obtained is given by

$$\Omega(L) = 2\phi_G = \int_\theta^{\frac{\pi}{2}} d\theta \int_0^\pi \sin\varphi d\varphi = \pi - 2\theta \tag{21}$$

as it was expected. Note that the geometric phase can be controlled by changing the device parameters, giving rise to different values of θ in (20). However, for changes different to the values given by (11), point B on the sphere is no longer reached by the electromagnetic state but an arbitrary point P (see Fig. 1). In this case, we still could close the circuit by a nonunitary transformation: for instance, by inserting a polarizer at the output plane of the channel waveguide, the orthogonal state $b(1,-1)^T$ can be always obtained. For showing this case, let us consider a polarizer which can be represented by the following matrix operator P

$$P = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \tag{22}$$

that is, its transmission axis forms an angle $-\pi/4$ with x-axis. When this polarizer acts on the state $|\varphi\rangle^T$, given by (3)–(5), it is obtained, after a strightforward calculation, the following state

$$|\varphi\rangle_B = \frac{\sqrt{2}}{2}\sin(\gamma d)\exp\left[\imath\left(\frac{\pi}{2} - \theta\right)\right]e^{\imath\alpha d}(1,-1) \tag{23}$$

where

$$\gamma = \frac{(\beta_{11} - \beta_{22})}{2\cos\theta}. \tag{24}$$

It corresponds to the evolution of the state $|\varphi\rangle$ along geodesic connecting the point P with B (see Fig. 1). Now, performing the above calculations for evaluating the geometric factors it can be easily proven that the geometric phase given by (21) is again obtained. Note that in this case the energy has decreased due to the nonunitary tranformation performed by the polarizer. We must indicate that the geometric phases generated by a coupling process between polarization states, caused by an anysotropic chiral medium, corresponds to the generalization of the geometric phase π obtained by a pure Faraday rotation [12], produced by an isotropic chiral medium acting on a linear polarization state. In our case a geometric phase equal to π is generated if $d' = \pi/\gamma$. Moreover, we also must stress that the same results could be obtained by using a magnetooptic bidirectional coupler [13], where the two waveguides play the role of TE and TM modes. In short, we have proven that a phase shift can have a fully topological origin, and its value depends on the path followed by the electromagnetic field in its projective Hilbert space.

An explicit example of device using these geometric effects corresponds to a reciprocal ferrite phase shifter developed by Boyd [14] (which could be implemented by magnetooptical waveguides). This device is a magnetically variable version of a rotatory-vane phase shifter described by Fox [15]. The Fox phase shifter is constituted by a fixed quarter-wave plate (QWP), a rotatable half-wave plate (HWP) and another fixed quarter-wave plate. An input field linearly polarized is launched into the first QWP (which is tilted at 45° with respect to the polarization direction of the input field).

The total delay is $(K + 2\theta)$, where K is the minimun phase delay through the device (that is, a dynamical phase) and θ is the tilt angle of the rotatable HWP. A full 360° geometrical range of phase shift can be obtained in analog form by varying θ through 180°. If we apply the topological results described above it can be shown starting from (9), after a long but straightforward calculation, that the geometric phase is $\phi_g = 2\theta$, and the dynamical phase is equal to K. This result can be easily proven by using the geometrical point of view (10). Boyd sustituted a ferrite rod with a rotatable transverse bias field for Fox's mechanically rotatable HWP, therefore the geometrical results are identical to those ones obtained for the Fox's device.

On the other hand, when θ is varied with an angular speed equal to ω_0 then a temporal phase shift is obtained (that is $(\omega_0 + \omega)t$, where ω is the central frequency of the input signal), which has a fully geometrical origin.

IV. Experimental System

We present two experimental arrangements for both obtaining and measuring the geometric phases generated by magnetooptic waveguides. The first of them corresponds to the device analyzed previously, the second one provides a simple arrangement for measuring geometric phases independently of the value of the dynamical phase (note that (23) only is not a function of γd (dynamical phase) for a discrete set of values given by (11)).

Let us consider an integrated Mach–Zehnder type interferometer, where each arm is a magnetooptic channel waveguide supporting TE and TM modes with different propagation constants. We start from an initial state at input plane $z = 0$, that is, the energy is launched in magnetooptic guides G_1 and G_2 (see Fig. 2(a)) by a splitting process, performed by isotropic dielectric channel waveguides, at point $(0, z_i)$, with initial intensity unity. Guide 2 is designed in such a way that β_- is reversed with respect to guide 1. By changing the waveguide's parameters of guides G_1 and G_2 (by electric and magnetic external fields), we can make measurements of intensity at point $(0, \tau)$ for different values of the geometric phase (see Fig. 2(a)). For making sure that the orthogonal state $b(1,-1)$ is reached a polarizer with transmission axis at 45° is located at plane τ; therefore, following the results of the Section III, the geometric factors generated are $(\pi/2 - \theta)$ (by guide 1) and $(\pi/2 + \theta)$ (by guide 2); now, taking into account (23) the total intensity at point $(0, \tau)$ is given by

$$I = \sin^2(\gamma d)\cos^2\theta. \tag{25}$$

Note that if γd takes the values $(2n + 1)\ \pi/2$ the intensity becomes

$$I = \cos^2\theta \tag{26}$$

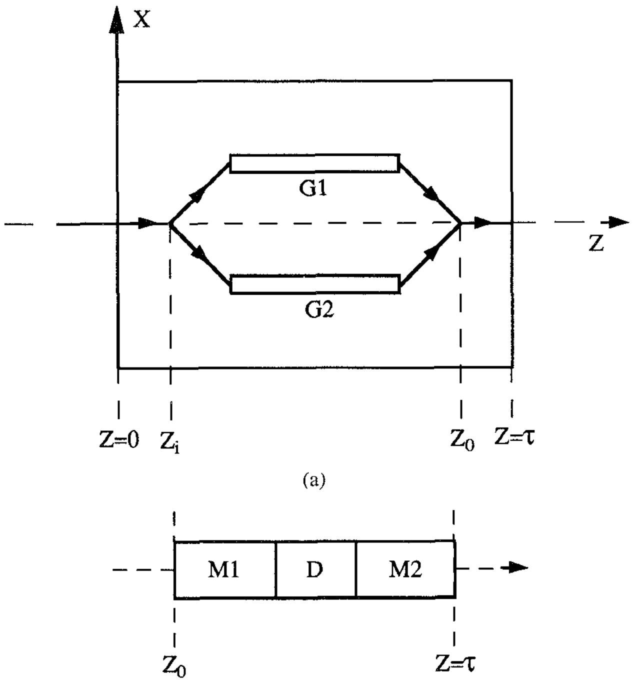

Fig. 2. (a) Schematic arrangement of an integrated interferometer whose arms are magnetooptic channel waveguides. (b) Schematic arrangement of a magnetooptic polarizer.

that is, the changes in the intensity only come from the geometric phase acquired by the state along the system.

Now, in order to obtain values of the intensity independent on the dynamical phase, we again consider the Mach–Zehnder interferometer. Nevertheless, in this case we consider $C = 0$ (the external magnetic field is switch off) in the guides G_1 and G_2; moreover we choose a length $z = \pi/2\gamma$ in a such a way that the state $|\varphi\rangle$ at the point $(0, z_0)$ is circularly polarized; in particular, since guide 2 is designed with β_- reversed with respect to guide 1, the light coming from arm 1 is LH at point $(0, z_0)$, and the light coming from arm 2 is RH at that point. Now, in the common path $z_0 - \tau$ the following consecutive channel waveguides are inserted (see Fig. 2(b)): first of them (M1) is a magnetooptic isotropic waveguide ($\beta_- = 0$) which provides a rotation $\rho = Cd$ (with d the length of the waveguide), the second one is a monomode waveguide (D), in particular only the TE mode can be propagated, and finally the third one (M2) is another magnetooptic isotropic waveguide with the external magnetic field reversed, in a such a way that a rotation $-\rho$ is produced. The three waveguides constitute a magnetooptic polarizer.

Let us consider that at point z_i the state $b(1,1)^T$ is launched; from a geometrical point of view (for this case we do not perform analytical calculations, in order to show the power of the geometrical approach), the guide 1 transports the initial state up to the north pole of the PS, and the second one one down the south pole (see Fig. 2). Now, the device along the path $z_i - \tau$ acts as a polarizer with transmission axis forming an angle ρ with respect to the TE-TM basis, therefore, the circular states reach the same point on the equator of the PS (AB effect on the PS). The global circuit is a slice one, thus the half solid angle subtended (geometric phase) is given by

$$\phi_G = \varphi - \frac{\pi}{2} \tag{27}$$

where φ is the azimuthal angle shown in Fig. 2. This angle is related to the physical parameters of the magnetooptic isotropic guides in the following way

$$\varphi = 2\rho = 2Cd. \tag{28}$$

Finally, since the intensity of the states interfering at point $(0, \tau)$ is the same, then the total intensity detected will be equal to

$$\begin{aligned} I &= \cos^2\left(\frac{\phi_G}{2}\right) = \cos^2\left(\frac{\varphi}{2} - \frac{\pi}{4}\right) \\ &= \cos^2\left(\rho - \frac{\pi}{4}\right) = \cos^2\left(Cd - \frac{\pi}{4}\right). \end{aligned} \tag{29}$$

Therefore, starting from the intensity measurements the geometric factor can be measured. Note that in this arrangment the dynamical phase has been cancelled.

In order to show the possible applications, we must stress that the geometric factor can be used for producing phase shifts (as it can be visualized from (18)). Moreover, when they are varied along the time a frequency shift is obtained; for instance, if we assume that $\rho = \omega_g t$ (i.e., the external magnetic field is varied linearly with the time), with ω_g much smaller that the carrier frequency ω then a frequency shift $\Delta\omega = \omega_g$ can be generated. In short temporal and spatial phase shift have a fully geometrical origin, that is, they are independent on the dynamical evolution. We must stress that some phase shifts can have a fully dynamical origin in whose case the geometrical phase given by (9) or (10) must be zero.

V. Conclusion

In conclusion, phase shifts generated by a coupling process in a magnetooptic channel waveguide have been presented. In particular the geometric phases produced by a 90°-rotor have been calculated by both geometrical and analytical methods. These phase factors have a geometrical origin, therefore they can be calculated by a geometrical approach, in this way, many of the conventional phase and frequency shifters can be explained by this topological approach, and likewise new shifters can be designed. On the other hand, an interferometric system has been described in order to both generate and measure these phases.

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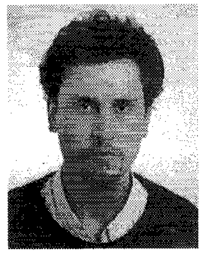

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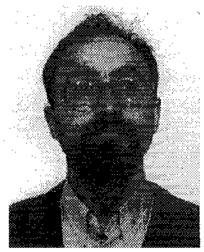

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